

# Learned Image Reconstruction for Interferometric Imaging

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September 28, 2023

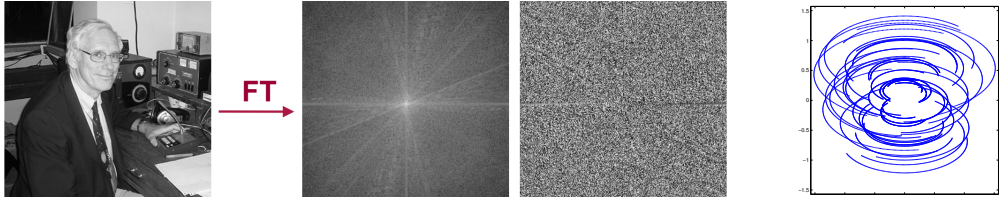
1. Interferometric Imaging
2. The Inverse Problem
3. Learned Image Reconstruction
4. Reconstruction for SPIDER
5. Reconstruction for Radio telescopes
6. Conclusion

# Interferometric Imaging

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# Interferometric Measurements

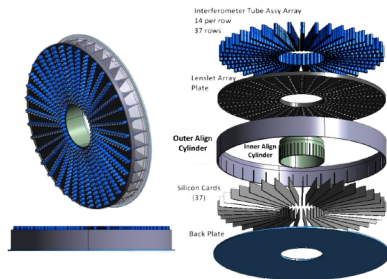
The **baselines** measure the **visibilities**  $y(u, v)$  which correspond to a Fourier measurement of the image:



*Credit: NRAO*

## Segmented Planar Imaging Detector for Electro-optical Reconnaissance

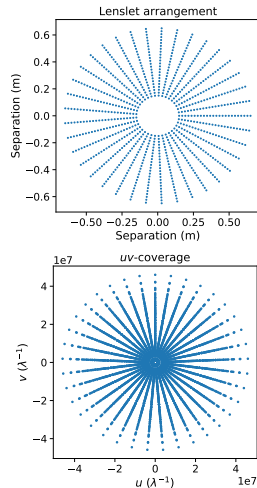
- ▷ SPIDER is new interferometric optical imaging device developed by UC Davis and Lockheed Martin.
- ▷ Lenslet array to measure multiple interferometric baselines and photonic integrated circuits (PICs) for **miniaturization**.
- ▷ Reduces weight, cost and power consumption of optical telescopes.



*Kendrick et al. 2013*

## Challenges for SPIDER:


- ▷ Sparse sampling of Fourier measurements. ( $M = 4440$ )
- ▷ Aliasing because of sampling distribution.
- ▷ Real-time reconstruction.




# Radio Telescopes


## SKA-mid – the SKA's mid-frequency instrument

The SKA Observatory (SKAO) is a next-generation radio astronomy facility that will revolutionise our understanding of the universe. It will have a uniquely distributed character: one observatory operating two telescopes on three continents. The two telescopes, named SKA-low and SKA-mid, will be observing the Universe at different frequencies. They are also called interferometers as they each comprise a large number of individual elements working together to form a single large telescope.







Location: South Africa




Frequency range:  
**350 MHz to 15.4 GHz**  
with a goal of 2.4 GHz




**197 dishes**  
(including 64 MeerKAT dishes)




Total collecting area:  
**33,000m<sup>2</sup>**




or  
**126 tennis courts**




Maximum distance between dishes:  
**150km**



Data transfer rate:  
**8.8 Terabits per second**



SKA-mid









Image quality of SKA-mid (left) versus the best current facility operating in the same frequency range, the Jansky Very Large Array (JVA) in the United States (right). SKA-mid's resolution will be 4x better than JVA.



**4x** the resolution



**5x** more sensitive



**60x** the survey speed


Compared to the JVA, the current best similar instrument in the world:


[www.skatelescope.org](http://www.skatelescope.org)

[@SKAO](#) [f SKA Observatory](#) [in SKA Observatory](#) [SKA Observatory](#) [@skaoobservatory](#)


## SKA-low – the SKA's low-frequency instrument

The SKA Observatory (SKAO) is a next-generation radio astronomy facility that will revolutionise our understanding of the universe. It will have a uniquely distributed character: one observatory operating two telescopes on three continents. The two telescopes, named SKA-low and SKA-mid, will be observing the Universe at different frequencies. They are also called interferometers as they each comprise a large number of individual elements working together to form a single large telescope.







Location: Australia




Frequency range:  
**50 MHz to 350 MHz**




**131,072 antennas** spread between **512 stations**




Total collecting area:  
**0.4km<sup>2</sup>**




Maximum distance between stations:  
**>65km**



Data transfer rate:  
**7.2 Terabits per second**



SKA-low









Image quality of SKA-low (left) versus the best current facility operating in the same frequency range, the LOFAR Frequency Array (LOFAR) in the Netherlands (right). SKA-low's resolution will be similar to LOFAR.



**25%** better resolution



**8x** more sensitive



**135x** the survey speed

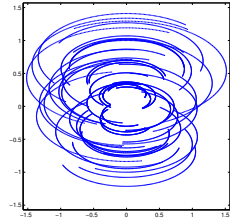
Compared to LOFAR Netherlands, the current best similar instrument in the world:

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## Challenges for Radio telescopes:

- ▷ Large amount of visibilities. ( $M \gg N$ )
- ▷ Visibility coverage depends on pointing, earth rotation, etc.





## The Inverse Problem

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# Inverse Problem

We can write this particular problem as an inverse problem

$$\mathbf{y} = \Phi \mathbf{x} + \mathbf{n},$$

with

- ▷  $\mathbf{y} \in \mathbb{C}^M$  – Non-uniformly distributed Fourier measurements at  $(u_j, v_j)$  for  $j = 0, \dots, M$
- ▷  $\Phi : \mathbb{R}^N \rightarrow \mathbb{C}^M$  – The forward operation of our measurement operator of the telescope
- ▷  $\mathbf{x} \in \mathbb{R}^N$  – The image to observe with  $N$  pixels
- ▷  $\mathbf{n} \in \mathbb{C}^M$  – Additive complex Gaussian noise

**Variational regularisation:** Find  $\mathbf{x}^*$  that satisfies

$$\mathbf{x}^* = \arg \min_{\mathbf{x} \in \mathbb{R}^N} \mathcal{L}(\Phi \mathbf{x}, \mathbf{y}) + \lambda \mathcal{R}(\mathbf{x}),$$

which we would typically solve in astronomical interferometry problems as

$$\mathbf{x}^* = \arg \min_{\mathbf{x} \in \mathbb{R}^N} \|\Phi \mathbf{x} - \mathbf{y}\|_2^2 + \lambda \|\Psi^\dagger \mathbf{x}\|_1,$$

where  $\Phi$  models the measurement operator and  $\Psi$  is typically a dictionary of wavelet bases.

# Learned Image Reconstruction

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Using deep learning to solve the inverse problem:

- ▷ Data-driven priors
- ▷ Higher reconstruction quality
- ▷ Better computational efficiency

# Learned Reconstruction Approaches

Learned solvers are approaches where the parameters  $\theta$  are Learned from training data.

**Learned regularisation:** Learn a regulariser  $\mathcal{R}_\theta$  and solve

$$\mathbf{x}^* = \arg \min_{\mathbf{x} \in \mathbb{R}^N} \mathcal{L}(\Phi \mathbf{x}, \mathbf{y}) + \lambda \mathcal{R}_\theta(\mathbf{x})$$

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**Learned Sequential models:** Learn a sequential Pseudo-inverse  $\Phi_\theta^\dagger$  such that

$$\mathbf{x}^* = \Phi_\theta^\dagger \mathbf{y}, \quad \Phi_\theta^\dagger = \mathbf{B}_\theta \circ \Phi^\dagger \circ \mathbf{C}_\theta$$

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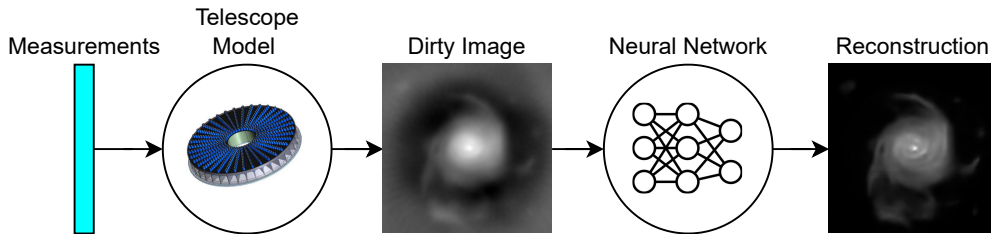
$$\mathbf{x}^* = \Phi_\theta^\dagger \mathbf{y}, \quad \Phi_\theta^\dagger = \mathbf{B}_\theta \circ \Phi^\dagger \circ \mathbf{C}_\theta$$

**Learned Unrolled Iterative Algorithms:** Learn how to iteratively update the reconstruction

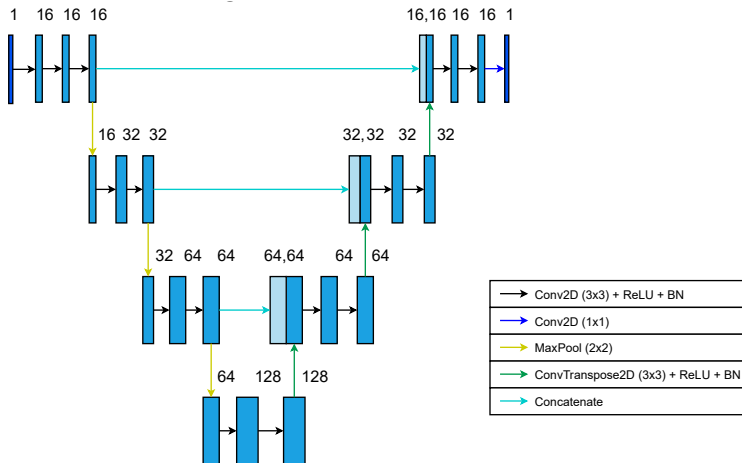
$$\mathbf{x}_{i+1} = \Lambda_\theta(\mathbf{x}_i, \nabla \mathcal{L}(\Phi \mathbf{x}_i, \mathbf{y})), \quad \text{for } i \text{ in } (0, \dots, N)$$



# Learned Post-processing

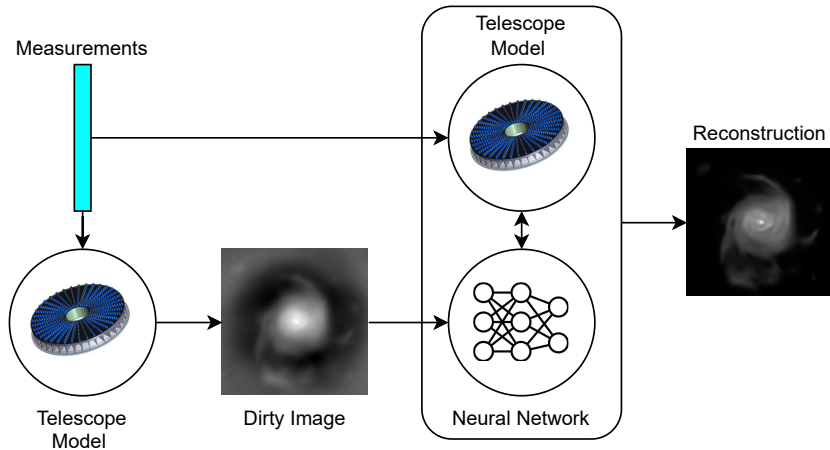


# Learned Post-processing Network (U-Net)

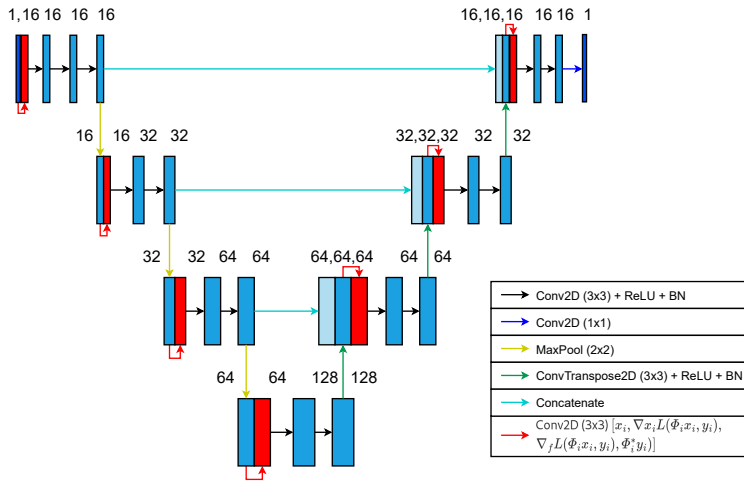


- ▷ Input: Dirty image
- ▷ Multi-resolution denoising

# Learned Unrolled Iterative approach



# GU-Net Architecture



Adding in the gradient of the data fidelity term to the U-Net structure.

For  $\ell_2$ -loss:

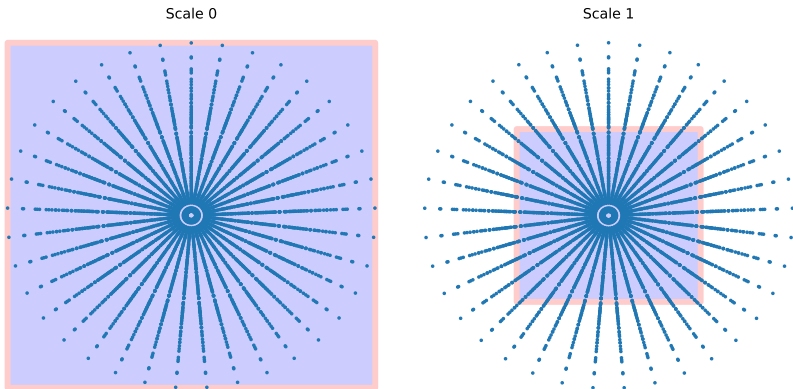
$$\mathcal{L} = \|\Phi \mathbf{x} - \mathbf{y}\|_{\ell_2}^2,$$

we calculate the gradient as follows:

$$\nabla \mathcal{L}(\Phi \mathbf{x}, \mathbf{y}) \propto \Phi^\dagger (\Phi \mathbf{x} - \mathbf{y})$$

# Learned Unrolled Iterative approach

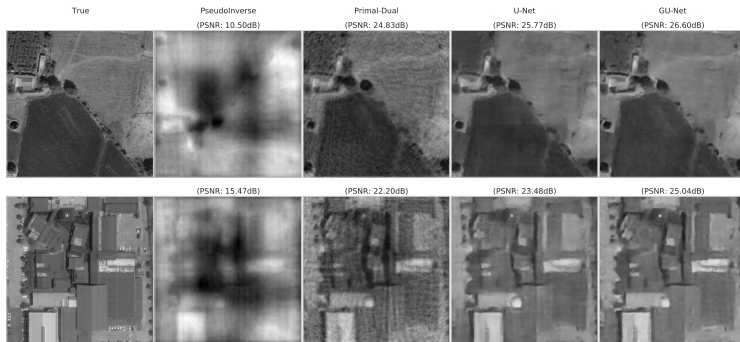
On each scale of the U-Net structure, we apply a subsampling of the measurement operator to use for the gradient of the data fidelity term.



## Reconstruction for SPIDER

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# Reconstructed SPIDER images



- ▷ Imaging time **speed-up of 50-600×** relative to classical approaches.
- ▷ Dramatic reduction in computational time opens up **real time imaging** with SPIDER for the first time.

# Computational Efficiency

Name	Operator evaluations	Average reconstruction time (ms)	Training time (mins)
Pseudo-inverse (1 GPU)	1	5.50	-
U-Net (1 GPU)	1	10.7	~30
GU-Net (1 GPU)	11*	42.1	~100
Primal-Dual (300its, 1 CPU)	600	$4.7 \times 10^4$	-

\*Refers to operator evaluation at the finest scale, which dominates the computational time of the GU-Net.

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## Reconstruction for Radio telescopes

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# Challenge in Radio Interferometry

**Challenge:** The visibility coverage is different for every observation

We compare:

▷ **True coverage (oracle):**

Actual visibility coverage, **full retraining** required for every observation.

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Different visibility coverage, **no retraining** required for every observation.

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Different visibility coverage, **no retraining** required for every observation.
- ▷ **Distribution of coverages:**  
Distribution of visibility coverages, **no retraining** required for every observation.

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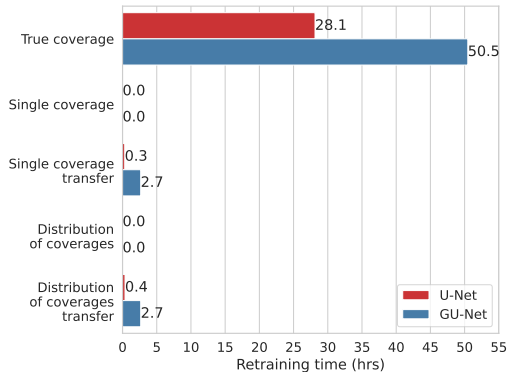
- ▷ **True coverage (oracle):**  
Actual visibility coverage, **full retraining** required for every observation.
- ▷ **Single coverage:**  
Different visibility coverage, **no retraining** required for every observation.
- ▷ **Distribution of coverages:**  
Distribution of visibility coverages, **no retraining** required for every observation.
- ▷ **Transfer Learning:**  
Fine-tuning through transfer learning using the **true coverage**, **small amount of retraining**.
  - ▷ **Single coverage transfer**
  - ▷ **Distribution of coverages transfer**

# Challenge in Radio Interferometry

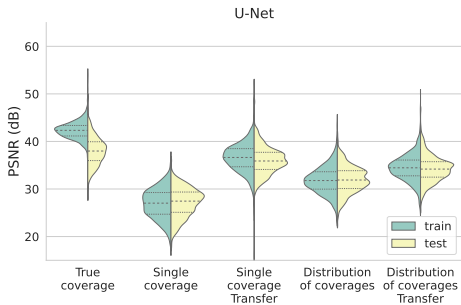
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- ▷ **Single coverage transfer**
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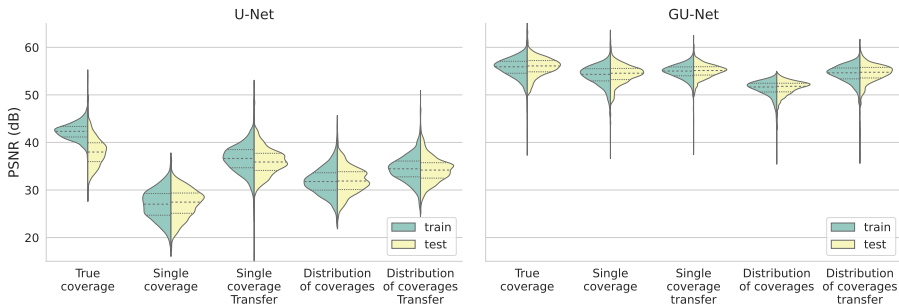


# Distribution of radio interferometric reconstruction quality



Reconstruction quality (PSNR  $\uparrow$ ) for different training strategies.

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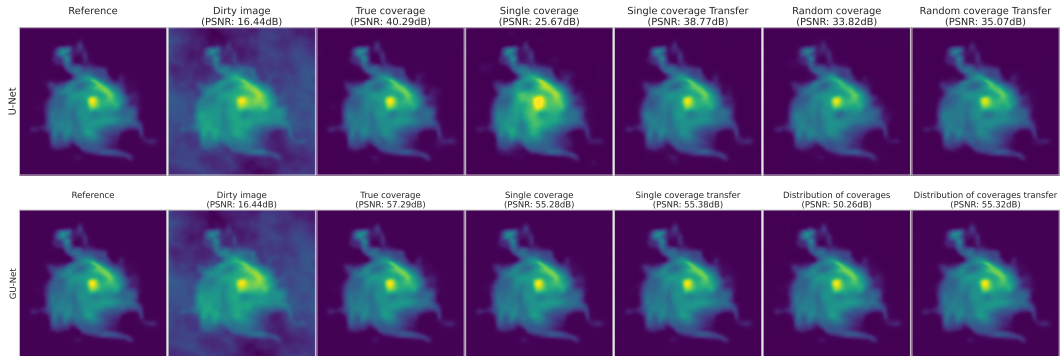


Reconstruction quality (PSNR  $\uparrow$ ) for different training strategies.

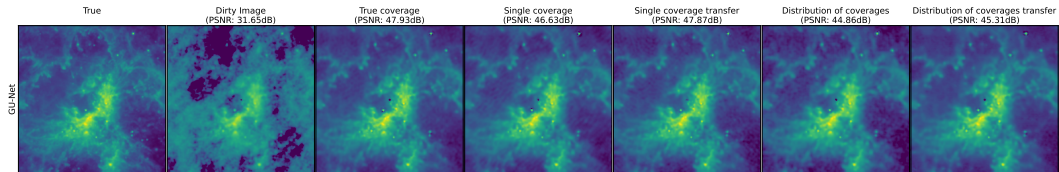
- ▷ Superior reconstruction quality by integrating physical model of instrument and more robust to measurement operator variability.
- ▷ Imaging time speed-up of 50-600 $\times$  relative to classical approaches.



# Reconstructed radio interferometric images



# Reconstructed radio interferometric images



- ▷ Full end-to-end learning for radio interferometric imaging with support for varying measurement operators for the first time.

## Conclusion

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- ▷ Imaging time **speed-up of 50-600×** relative to classical approaches.
- ▷ **Superior reconstruction quality** by integrating physical model of instrument
- ▷ **Full end-to-end learning** for radio interferometric imaging with support for **varying measurement operators** for the first time.

## Papers:

Mars et al. 2023, "Learned Interferometric Imaging for the SPIDER Instrument",  
arXiv:2301.10260

Mars et al. (in prep.), "Learned radio interferometric imaging for varying visibility coverages"